- Axiometic Approach of Probability

- Probability (of an event E) $= \dfrac{\text{no. of favourable outcomes}}{\text{total no. of outcomes}}$

$= P(E)$

$\boxed{0 \le P(E) \le 1}$

- Outcomes = Results.

- Random Experiment : an experiment which has two or more well defined outcomes, but we can not Predict the next outcome in advance.

e.g. Tossing a fair coin
Throwing a fair Die

- Sample Space : Set of all possible outcomes in a random experiment.

e.g. Random Experiment → Tossing a Die.
Sample Space (S) → $S = \{1, 2, 3, 4, 5, 6\}$

- Event (E): what we want (subset of sample space)

e.g. Random Experiment → Tossing a Die
Sample Space $S = \{1, 2, 3, 4, 5, 6\}$

$\underbrace{\text{We want PRIME NUMBERS}}_{\text{Event (E)}}$ $E = \{2, 3, 5\}$

$P(\text{Prime}) = P(E) = \dfrac{3 \leftarrow \{2,3,5\}}{6 \leftarrow \{1,2,3,4,5,6\}}$

$= \frac{1}{2} = 0.5$

Some Important Sample Spaces

- **Tossing a fair Coin**

  $S = \{H, T\}$ = Sample space

  no. of elements in $S = n(S) = 2 = 2^1$

- **Tossing 2 coins / (Tossing a Coin Twice)**

  $S = \{HH, HT, TH, TT\}$

  $n(S) = 4 = 2^2$

- **Tossing 3 Coins / (Tossing a Coin Thrice)**

  $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

  $n(S) = 8 = 2^3$

- **Roll a Die (Throw a Die)**

  $S = \{1, 2, 3, 4, 5, 6\}$

  $n(S) = 6$

- **Roll 2 Dice (Roll a Die twice)**

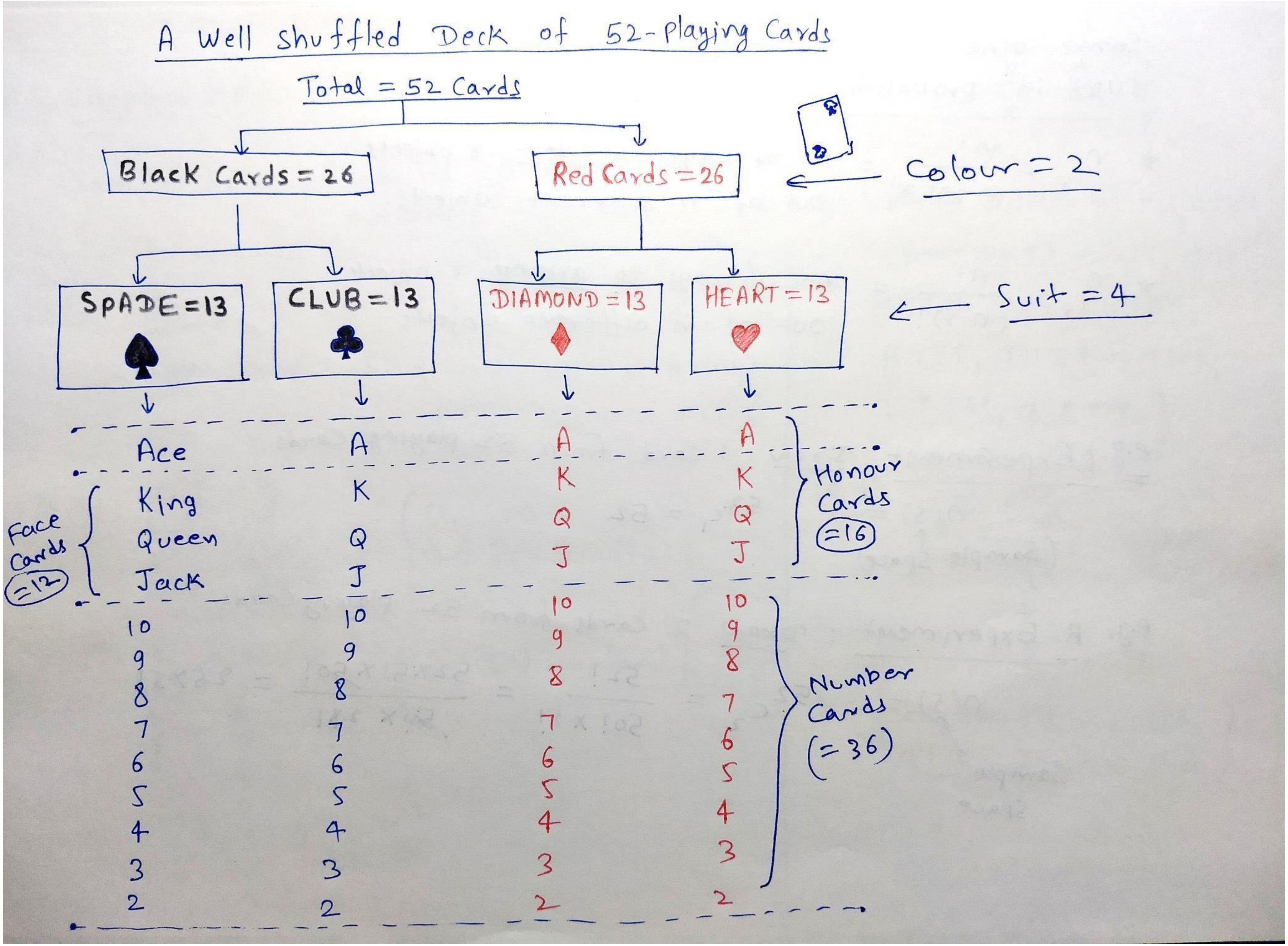
$$S = \left\{ \begin{matrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,5) & (4,4) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{matrix} \right\}$$

  $n(S) = 36$

- **Toss a Coin & Roll a Die**

$$S = \left\{ \begin{matrix} (H,1) & (H,2) & (H,3) & (H,4) & (H,5) & (H,6) \\ (T,1) & (T,2) & (T,3) & (T,4) & (T,5) & (T,6) \end{matrix} \right\}$$

  $n(S) = 2 \times 6 = 12$


A Well Shuffled Deck of 52-Playing Cards
Total = 52 Cards
Black Cards = 26
Red Cards = 26
Colour = 2
SPADE = 13
CLUB = 13
DIAMOND = 13
HEART = 13
Suit = 4
Ace
King
Queen
Jack
10
9
8
7
6
5
4
3
2
A
K
Q
J
Face Cards (=12)
Honour Cards (=16)
Number Cards (=36)

Some other important tools used in probability.

★ ${}^{n}C_{r} = \dfrac{n!}{(n-r)!\, r!}$ = no. of ways to select r objects out of n-different objects.

★ ${}^{n}P_{r} = \dfrac{n!}{(n-r)!}$ = no. of ways to arrange r-objects out of n-different objects

e.g. R.Experiment: Draw 1 card from 52 playing Cards

$n(S) = {}^{52}C_{1} = 52$

(Sample Space)

e.g. R. Experiment. : Draw 2 cards from 52 playing Cards

$n(S) = {}^{52}C_{2} = \dfrac{52!}{50! \times 2!} = \dfrac{52 \times 51 \times \cancel{50!}}{\cancel{50!} \times 2 \times 1} = 26 \times 51$

Sample Space

Exercise **14.1**

Sample Space : "Set of all possible outcomes"

Q.1 A coin → 3 times toss

H H H
T T T

Sample space = $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

Q.2

1st time | 2nd time
1 | 1
2 | 2
3 | 3
4 | 4
5 | 5
6 | 6

$$S = \begin{Bmatrix} (1,1) & (1,2) & \ldots\ldots & (1,6) \\ (2,1) & (2,2) & \ldots & (2,6) \\ \vdots & \vdots & & \vdots \\ (6,1) & (6,2) & \ldots & (6,6) \end{Bmatrix}$$

$n(S) = 36$

Q.3

H H H H
T T T T

$S = \{HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THHT, HTTH, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT\}$

Q.4

H
T

1
2
3
4
5
6

$S = \{H1, H2, H3, \ldots H6, T1, T2, T3, \ldots T6\}$

Q.5

H → 1, 2, 3, 4, 5, 6
T

$S = \{T, H1, H2, H3, H4, H5, H6\}$

Q.6

X	Y
$2B \to B_1, B_2$	$1B \to B_3$
$2G \to G_1, G_2$	$3G \to G_3, G_4, G_5$

I → Room ✓

II → person. ✓

$$S = \{XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5\}$$

Q.7

Red (R) White (W) Blue (B)

$$S = \{R1, R2, R3 \dots R6, W1, W2, W3 \dots W6, B1, B2, B3, \dots B6\}$$

Q.8

(i) B → Boy

G → Girl

$S = \{BB, BG, GB, GG\}$

(ii) $S = \{0, 1, 2\}$

number of Girls

0, 1, 2, 3, ...

Q.9

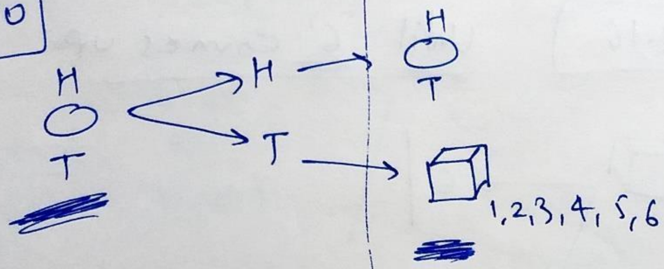
R W W

Bag

(without replacement)

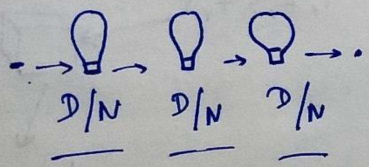
$S = \{RW, WR, WW\}$

Q.10



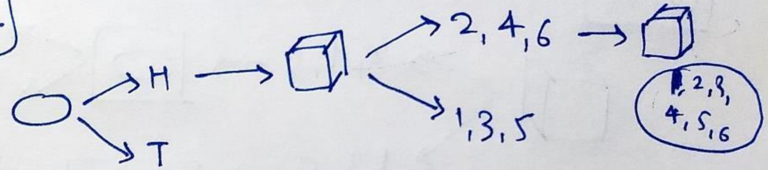
S = { HH, HT, T1, T2, T3, T4, T5, T6 }

Q.11



S = { DDD, DDN, DND, NDD, DNN, NDN, NND, NNN }

Q.12



S = { T, H1, H3, H5, (H,2,1), (H,2,2) ... (H,2,6), (H,4,1), (H,4,2) .... (H,4,6), (H,6,1), (H,6,2), ... (H,6,6) }

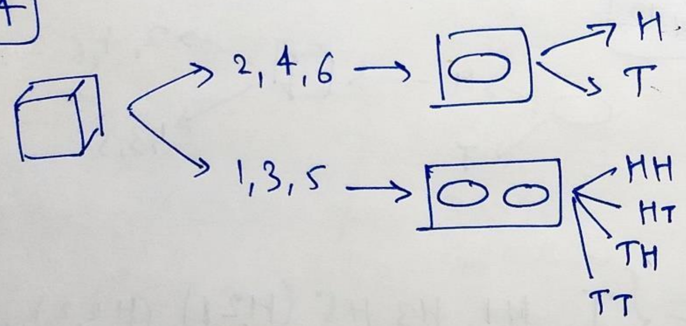
Q.13

1, 2, 3, 4

without replacement

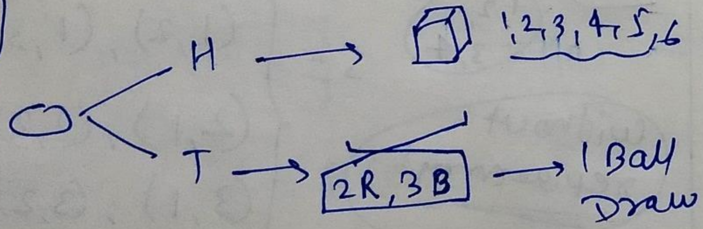
S = { (1,2), (1,3), (1,4),
(2,1), (2,3), (2,4)
(3,1), (3,2), (3,4)
(4,1), (4,2), (4,3) }

Q. 14



$S = \{$ 2H, 2T, 4H, 4T, 6H, 6T,
1HH, 1HT, 1TH, 1TT
3HH, 3HT, 3TH, 3TT
5HH, 5HT, 5TH, 5TT $\}$

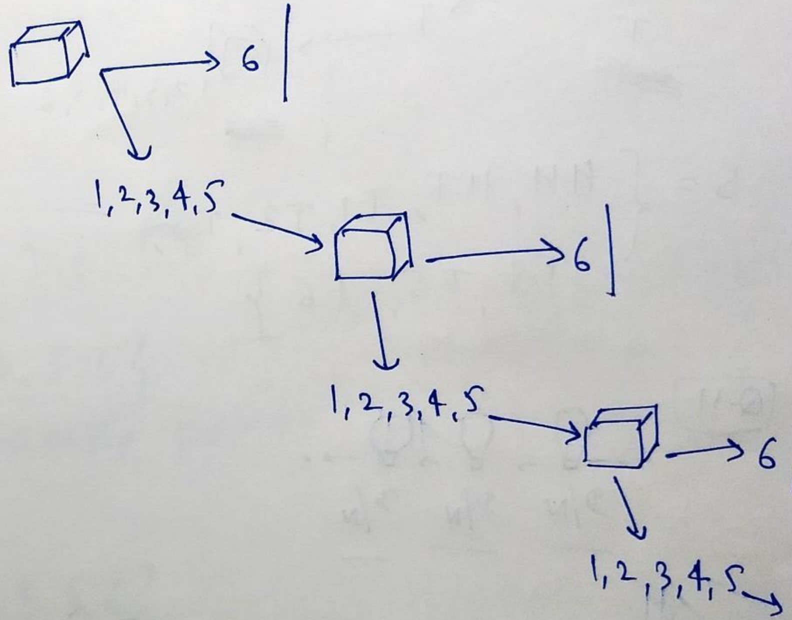
Q. 15



$S = \{$ H1, H2, H3, H4, H5, H6,
TR_1, TR_2, TB_1, TB_2, TB_3 $\}$

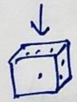
Q. 16

Until '6' comes up.



$S = \{$ 6,
(1, 6), (2, 6), (3, 6), (4, 6), (5, 6),
(1, 1, 6), (1, 2, 6), (1, 3, 6) . . . (1, 5, 6),
(2, 1, 6), (2, 2, 6), (2, 3, 6) . . . (2, 5, 6)
. . . . . $\}$

Event (E): 'what we want' / subset of sample space (S)

e.g. Random Experiment: Rolling a Die 

Sample Space $= S = \{\underbrace{1, 2, 3, 4, 5, 6}_{\text{Sample points}}\}$ $\rightarrow$ (No. of Sample points $= 6$)

Event: we want prime No.

$$E = \{2, 3, 5\} \subset \{1, 2, 3, 4, 5, 6\}$$

$\boxed{E \subseteq S}$ $\qquad P(E) = P(\text{Prime}) = \frac{3}{6} = \frac{1}{2}$

Note: $0 \leq P(E) \leq 1$

$P(E) = 1 \Rightarrow$ Chance $= 100\% \Rightarrow E =$ Sure Event

$P(F) = 0 \Rightarrow$ Chance $= 0\% \Rightarrow F =$ Impossible Event

Different Types of Events:

Probability ↔ Set

① **Impossible Event**: $P(E) = 0 = P(\phi)$ ϕ = empty set = $\{\ \}$

② **Sure Event**: $P(E) = 1$

③ **Simple Event**: Event which has only one sample point.

e.g. Tossing a coin. → $S = \{H, T\}$

Event 'E' = getting Head

Simple event. → $E = \{H\}$

Start to link Set & Probability

④ **Compound Event**

(more than one sample point)

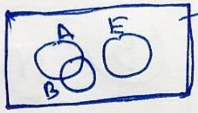
e.g. Experiment → Rolling a Die

$S = \{1, 2, 3, 4, 5, 6\}$

Event E = Even no. = $\{2, 4, 6\}$

Algebra of Events.

Event → set



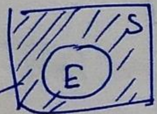
→ U = Sample Space = S

① Complementary Event

Event → E

Complementary Event = $E' = \overline{E} = E^c$ → not 'E'

$\boxed{E' = S - E}$



e.g. Roll a Die

$S = \{1, 2, 3, 4, 5, 6\}$

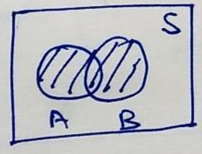
E = no. less than 3

$E = \{1, 2\}$

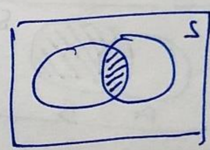
$E' = \{3, 4, 5, 6\}$

② Event 'A' or 'B' = $A \cup B$

③ Event 'A' and 'B' = $A \cap B$



$A \cup B$



$A \cap B$
(Common part)

e.g. Rolling a Die

$S = \{1, 2, 3, 4, 5, 6\}$

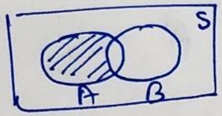
E = no. less than '3' = $\{1, 2\}$

F = odd no. = $\{1, 3, 5\}$

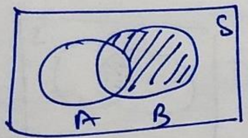
$E \cup F$	$E \cap F$
$\{1, 2, 3, 5\}$	$\{1\}$
$P(E \cup F) = \frac{4}{6}$	$P(E \cap F) = \frac{1}{6}$

④ Event 'A but not B'

$A - B = A \cap \underline{B'}$



Event 'B but $\underline{\text{not } A}$' $= B - A$
$= B \cap A'$



$\underline{\underline{\text{e.g.}}}$ Roll a Die 

$S = \{1, 2, 3, 4, 5, 6\}$ ✓

$E =$ No. less than $3 = \{1, 2\}$

$F =$ odd no. $= \{1, 3, 5\}$

↳ Find prob. of getting odd no. $\underline{\text{but not}}$ less than '3'. $\Rightarrow \boxed{F - E} = \{\underline{1, 3, 5}\} - \{1, 2\}$

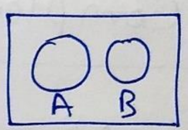
$= \{3, 5\}$

$P(F - E) = \frac{2}{6}$

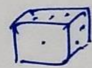
⑤ $\underline{\text{Mutually Exclusive Events}}$.

which have nothing in common.

Disjoint Sets



$\underline{\underline{A \cap B}} = \{ \} = \phi$

$\underline{\underline{\text{e.g.}}}$ Rolling a Die 

$S = \{1, 2, 3, 4, 5, 6\}$

$E = \{1, 2\}$ $H = \{6\}$

$F = \{1, 3, 5\}$

E & F → not mutually exclusive Events.

E & H → MEE ✓

F & H → MEE ✓

⑥ Exhaustive Events:

$E_1, E_2, \ldots, E_n$ are said to be Exhaustive Events if atleast one of them necessarily occurs whenever experiment is performed.

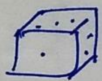
mathematically.

$$E_1 \cup E_2 \cup E_3 \cup \cdots \cup E_n = \underset{\text{Sample space}}{S}$$

⑦ Mutually Exclusive & Exhaustive Events:

E & F $\rightarrow$ $E \cap F = \phi$
$\quad\rightarrow$ $E \cup F = S$

A & B $\rightarrow$ $A \cap B = \phi$
$\quad\rightarrow$ $A \cup B = S$

e.g. Rolling a Die. 

$S = \{1, 2, 3, 4, 5, 6\}$ ✓

$E = \{1, 3, 5\}$ $\quad$ $A = \{2, 3\}$

$F = \{2, 4, 6\}$ $\quad$ $B = \{1, 4, 5, 6\}$

$C = \{5\}$

✓ E & F $\rightarrow$ $E \cup F = \{1, 2, 3, 4, 5, 6\} = S$

✓ A & B $\rightarrow$ $A \cup B = \{1, 2, 3, 4, 5, 6\} = S$

✗ E & A $\rightarrow$ $E \cup A = \{1, 2, 3, 5\} \neq S$

✓ E, F, A $\rightarrow$ $E \cup F \cup A = \{1, 2, 3, 4, 5, 6\} = S$

Exercise 14.2

Sure Event

Impossible Event

Simple Event

Compound Event

Complementary Event

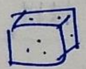
$A \cup B$ 'A or B'

$A \cap B$ 'A & B'

$A - B$ 'A but not B'

Mutually Exclusive Events.

Exhausitive Events.

Q.1  $S = \{1, \underline{2}, 3, \underline{4}, 5, \underline{6}\}$

$E = \{4\}$ $F = \{2, 4, 6\}$

Empty

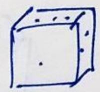
$\{\ \}$

Mutually Exclusive Events: $E \cap F = \phi$

$E \cap F = \{\underline{4}\} \cap \{2, \underline{4}, 6\} = \{4\} \neq \phi$

Common

E & F are not MEE

Q.2  Die

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2, 3, 4, 5, 6\}$

$B = \{\ \} = \phi$

$C = \{3, 6\}$

$D = \{1, 2, 3\}$

$E = \{6\}$

not F

$F = \{3, 4, 5, 6\}$, $F' = \{1, 2\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$A \cap B = \phi = \{\ \}$

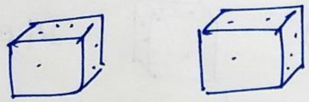
$A - C = \{1, 2, 3, 4, 5, 6\} - \{3, 6\}$

$= \{1, 2, 4, 5\}$

$E \cap F' = \{6\} \cap \{1, 2\}$

$= \phi = \{\ \}$

Q.3



$$S = \begin{Bmatrix} (1,1) & (1,2) & \cdots & (1,6) \\ (2,1) & (2,2) & \cdots & (2,6) \\ \vdots & \vdots & & \vdots \\ (6,1) & (6,2) & \cdots & (6,6) \end{Bmatrix}$$

A : sum is greater than 8

$\downarrow$

9, 10, 11, 12

A = {(6,3), (5,4), (4,5), (3,6), (6,4), (5,5), (4,6), (6,5), (5,6), (6,6)}

B: 2 occurs on either Die.

B = {(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (1,2), (3,2), (4,2), (5,2), (6,2)}

~~Q.4~~ C : Sum at least 7 & multiple of 3.

Sum $\to$ 7, 8, 9, 10, 11, 12

C = {(6,3), (5,4), (4,5), (3,6), (6,6)}

Q.4

H H H
T T T

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

A : 3 H's = {HHH}

B : 2 H's & 1 T = {HHT, HTH, THH}

C : 3 T's = {TTT}

D: H on the first coin = {HHH, HHT, HTH, HTT}

(i) Mutually Exclusive Events $E \cap F = \phi$

Disjoint

A & B, | A & C, | B & C | C & D

$A \cap B = \phi$

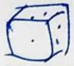
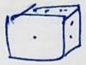
(ii) Simple events. only one element

A, C

(iii) Compound Events. more than one ~~less~~ elements

B, D

Q.5 Self

Q.6  

$$S = \begin{Bmatrix} (1,1) & (1,2) & \dots & (1,6) \\ (2,1) & (2,2) & \dots & (2,6) \\ \vdots & \vdots & & \vdots \\ (6,1) & (6,2) & \dots & (6,6) \end{Bmatrix}$$

A = even no. of first die

B = odd no. of first die

C = Sum ≤ 5 $\rightarrow$ Sum $\rightarrow$ 5, 4, 3, 2

$$A = \begin{Bmatrix} (2,1), (2,2) \dots (2,6) \\ (4,1), (4,2) \dots (4,6) \\ (6,1), (6,2) \dots (6,6) \end{Bmatrix}$$

$$B = \begin{Bmatrix} (1,1) & (1,2) & \dots & (1,6) \\ (3,1) & (3,2) & \dots & (3,6) \\ (5,1), & (5,2). & \dots & (5,6) \end{Bmatrix}$$

$$C = \begin{Bmatrix} (4,1), (3,2), (2,3), (1,4) \\ (3,1), (2,2), (1,3), (1,2), (2,1), (1,1) \end{Bmatrix}$$

Q.7.

(i) A & B are mutually exclusive events. (True)

(ii) A & B are mutually exclusive & exhaustive events.

$A \cap B = \phi$ ✓, $A \cup B = S$

(rows 1, 3, 5) (rows 2, 4, 6)

(True)

(iii) $A = B'$ True

(iv) A & $C \rightarrow$ mutually exclusive

$A \cap C \neq \phi$ False

(V) A & B' are mutually exclusive. False

$A \cap B' \neq \phi$ By (iii) part

$A \cap A \neq \phi$

$A \neq \phi$

(VI) A', B', C are mutually exclusive & ~~exclusi~~ Exhaustive

False

~~$A' \cap B' \cap C = \phi$~~

~~$\Rightarrow B \cap A \cap A \cap C = \phi$~~

only for 2

$A = B'$

$A' = (B')'$

$A' = B$

$A' \cap B' = B \cap A = \phi$

$A' \cap C = B \cap C \neq \phi$ Not MEE

$B' \cap C$

Probability : Axiometic Approach to Probability

- Probability of the Event (E)

$$= P(E) = \frac{\text{no. of outcomes favourable to } E}{\text{total possible outcomes}}$$

- Conditions:

S (Sample Space)

(i)	$0 \leq P(E_1) \leq 1$
(ii)	$P(E_1) + P(E_2) + P(E_3) \ldots + P(E_n) = P(S) = 1$

$$S = \{a_1, a_2, a_3, \ldots, a_n\}$$

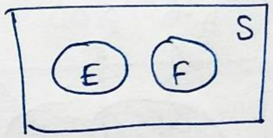
$E_1 = \{a_1\}$ Simple Event

$E_2 = \{a_2\}$ simple —

$\vdots$

$E_n = \{a_n\}$

- Mutally Exclusive Events: E & F

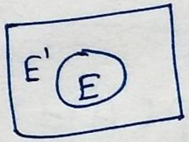


$$E \cap F = \phi = \{\}$$

$$P(E \cap F) = 0$$

$$P(\text{both } E \,\&\, F) = 0$$

- Complementary Events.

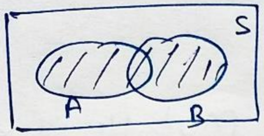


$$E \cup E' = S$$

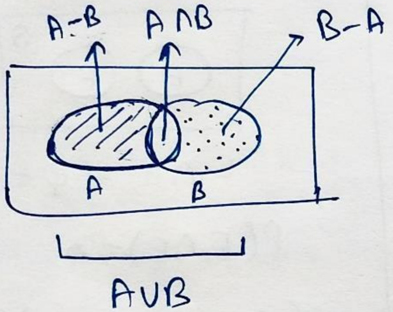
$$P(E \cup E') = P(S) = 1$$

$$\Rightarrow P(E) + P(E') = 1$$

$$\boxed{P(E') = 1 - P(E)}$$

- 

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- 

$$P(A \cup B) = P(A-B) + P(A \cap B) + P(B-A)$$

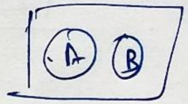
$$P(A) = \underline{P(A-B)} + P(A \cap B)$$

$$P(A \cap B') = P(A-B) = P(A) - P(A \cap B)$$

$$P(A' \cap B) = P(B-A) = P(B) - P(A \cap B)$$

- If A & B are mutually exclusive events then:

$A \cap B = \phi$



$$P(A \cup B) = P(A) + P(B) - \underline{P(A \cap B)}$$

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

0

- <u>De Morgan's Law</u>:

$\Rightarrow P((A \cup B)') = P(A' \cap B')$

$= 1 - P(A \cup B)$

Question

$\Rightarrow P((A \cap B)') = P(A' \cup B')$

$= 1 - P(A \cap B)$

Exercise - 14.3

Q.1 Sample Space $= S = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$

(a) $0 \le P(\omega_i) \le 1$ ✓

$P(\omega_1) + P(\omega_2) \cdots + P(\omega_7)$

$\sum_{i=1}^{7} P(\omega_i) = 0.1 + 0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6$

$= 1.00$

$\sum P(\omega_i) = 1$ ✓

Valid

(b) $\frac{1}{7}$ $\frac{1}{7}$ $\frac{1}{7}$ $\frac{1}{7}$ $\frac{1}{7}$ $\frac{1}{7}$ $\frac{1}{7}$

$0 \le \underbrace{P(\omega_i)}_{\frac{1}{7}} \le 1$ ✓

Valid.

$\sum_{i=1}^{7} P(\omega_i) = 7 \times \frac{1}{7} = 1$ ✓

(c) 0.1 0.2 0.3 0.4 0.5 0.6 0.

$0 \le P(\omega_i) \le 1$ ✓

$\sum_{i=1}^{7} P(\omega_i) = 2.8 \ne 1$ ✗

not valid

(d) −0.1 0.2 0.3 0.4 −0.2 0.1 0.3

Negative

$P(\omega_i) = -0.1$ Not possible.

Not valid.

(e) $\frac{1}{14}$ $\frac{2}{14}$ $\frac{3}{14}$ $\frac{4}{14}$ $\frac{5}{14}$ $\frac{6}{14}$ $\frac{15}{14}$

$\frac{15}{14} > 1$ $P(\omega_7) > 1$

$0 \le P(\omega_i) \le 1$

Not valid.

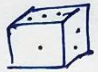
Q.2

$\underset{T}{\overset{H}{\bigcirc}}$ $\underset{T}{\overset{H}{\bigcirc}}$

$S = \{HH, HT, TH, TT\}$

P(atleast one tail)

$= \frac{3}{4} \leftarrow HT, TH, TT$

Q.3 a die 

$S = \{1, 2, 3, 4, 5, 6\} \leftarrow$ total

(i) $P(\text{Prime no.}) = \frac{3 \leftarrow 2,3,5}{6 \leftarrow \text{total}}$

$= \frac{1}{2}$

(ii) $P(\text{number} \ge 3) = \frac{4 \leftarrow 3,4,5,6}{6 \leftarrow \text{total}}$

$= \frac{2}{3}$

(iii) $P(\text{no.} \le 1) = \frac{1 \leftarrow \{1\}}{6}$

(iv) $P(\text{no.} > 6) = \frac{0}{6} \leftarrow$ no. $\not> 6$

impossible event

$= 0$

(v) $P(\text{no.} < 6) = \frac{5}{6} \leftarrow \{1, 2, 3, 4, 5\}$

Q.4 A card is selected from 52 cards.

any one of the 52 cards can come here.

(i) Sample space = Set of all Cards.

$n(S) = 52$

(ii) $P(\text{ace of spade}) = \dfrac{1}{52}$ ← total

(iii) $P(\text{ace}) = \dfrac{4}{52} = \dfrac{1}{13}$

$P(\text{Black card}) = \dfrac{26}{52} = \dfrac{1}{2}$

Q.5 Coin Die

1

6

1, 2, 3, 4, 5, 6

$\text{Value}_1 + \text{Value}_2$

(i) Sum = 3

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$P(\text{Sum} = 3) = \dfrac{1}{12}$ ← (1,2) ← total

(ii) Sum = 12

$P(\text{Sum} = 12) = \dfrac{1}{12}$ ← (6,6) ← total.

Q.6 4 Men , 6 Women

$\underbrace{\text{4 Men , 6 Women}}_{\text{1 person}}$

$$P(\text{woman}) = \frac{6 \leftarrow \text{women}}{10 \leftarrow \text{total}}$$

$$= \frac{3}{5}.$$

Q.7

$$\underset{T}{\overset{H}{\bigcirc}} \quad \underset{T}{\overset{H}{\bigcirc}} \quad \underset{T}{\overset{H}{\bigcirc}} \quad \underset{T}{\overset{H}{\bigcirc}}$$

S = { HHHH, HHHT, HHTH, HTHH, THHH,
HHTT, HTHT, THHT, HTTH, THTH,
TTHH, HTTT, THTT, TTHT, TTTH,
TTTT }

$H \rightarrow +1$ ₹

$T \rightarrow -1.5$ ₹

Different types of amounts

HHHH → 4

HHHT → 1.5 ($1 + 1 + 1 - 1.5$)

HHTT → -1 ($\underline{1\ \ 1}$ $\underline{-1.5\ -1.5}$; $2 - 3$)

HTTT → -3.5 ($1 - 1.5 \times 3$)

TTTT → -6 ($-(1.5) \times 4$)

Different amounts

$= \{4, 1.5, -1, -3.5, -6\}$

$$P(\text{amt} = 4) = \frac{1 \leftarrow \text{HHHH}}{16}$$

$P(1.5) = \frac{4}{16} = \frac{1}{4}$

HHHT
HHTH
HTHH
THHH

$P(-1) = \frac{6}{16} = \frac{3}{8}$

(HHTT
⋮)

$P(-3.5) = \frac{4}{16} = \frac{1}{4}$

(HTTT, ...)

$P(-6) = \frac{1}{16}$

TTTT

$\boxed{Q.8}$ H/T H/T H/T

Sample Space $= S = \{\underline{HHH}, \underline{HHT, HTH, THH}, \underline{HTT, THT, TTH}, \underline{TTT}\}$

$n(S) = 8$

(i) $P(3H) = \frac{1}{8}$ ← HHH

(ii) $P(2H) = \frac{3}{8}$

(iii) $P(\text{atleast } 2H) = \frac{\overset{2H}{3} + \overset{3H}{1}}{8} = \frac{4}{8} = \frac{1}{2}$

↓

2H या 3H

(iv) $P(\text{atmost } 2H) = \frac{\overset{0H}{1} + \overset{1H}{3} + \overset{2H}{3}}{8} = \frac{7}{8}$

0H 1H 2H

↓

TTT

(V) $P(\text{no head}) = \frac{1}{8}$ ← TTT

(VI) $P(3T) = \frac{1}{8}$ ← TTT

(VII) $P(\text{exactly } 2T) = \frac{3}{8}$ ← TTH, HTT, THT

(VIII) $P(\text{No tail}) = \frac{1}{8}$ ← HHH

(IX) $P(\text{atmost } 2\,T) = \frac{1+3+3}{8} = \frac{7}{8}$.

0 T (HHH) 1T 2T

Q.9 Event 'A'

$P(A) = \frac{2}{11}$

total (A)

$P(\text{not } A) = P(A') = 1 - \frac{2}{11}$

(Complementary event)

$= \frac{9}{11}$

Q.10 ASSASSINATION

Vowels → {A, A, A, I, I, O} → 6

Consonants → {S, S, S, S, N, T, N} → 7

total = 13

$P(\text{vowel}) = \frac{6}{13}$ ✓

$P(\text{consonant}) = \frac{7}{13}$ ✓

Q.11

Box

1, 2, 3, ... , 20 → 6 no.

"Committee has special 6 numbers" ← Prize

→ (2, 7, 8, 10, 13, 15)

Total No. of ways to select 6 numbers out of $20 = {}^{20}C_6$

$$= \frac{20!}{14! \times 6!} = \frac{\cancel{20} \times 19 \times \cancel{18} \times 17 \times \cancel{16}^{8} \times 15 \times \cancel{14!}}{\cancel{14!}\ \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}$$

$= 19 \times 17 \times 8 \times 15$

$= 38760$

To win the prize money a person has one way to select only those special numbers.

Favourable cases = 1

$$P = \frac{1 \leftarrow \text{win}}{38760 \leftarrow \text{total}}$$

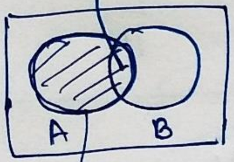
Q.12

Consistently defined → Yes ? / No ?

(i) $P(A) = 0.5$

$P(B) = 0.7$

$P(A \cap B) = 0.6$ ← common.



0.5

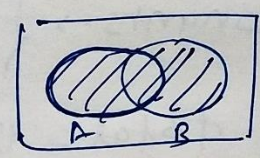
$P(A) < P(A \cap B)$

∴ Not Valid.

(ii) $P(A) = 0.5$

$P(B) = 0.4$

$P(A \cup B) = 0.8$



Yes

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.5 + 0.4 - 0.8 = 0.1$ ✓

Q.13 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\boxed{P(A \cup B) + P(A \cap B) = P(A) + P(B)}$$

	P(A)	P(B)	P(A∩B)	P(A∪B)
(i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	$\boxed{\frac{7}{15}}$
(ii)	0.35	$\boxed{0.5}$	0.25	0.6
(iii)	0.5	0.35	$\boxed{0.25}$	0.6

$\underbrace{P(A) + P(B)}_{+} = \underbrace{P(A\cap B) + P(A\cup B)}_{+}$

$\frac{1}{3} + \frac{1}{5} = \frac{1}{15} + x$

$\frac{8}{15} = \frac{1}{15} + x$

Q.14 $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{5}$

$P(A \text{ or } B)$ Mutually Exclusive events.

$\downarrow$

$\boxed{P(A \cap B) = 0}$

$P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{5} + \frac{1}{5} \qquad 0$

$= \frac{4}{5}$

Q.15 $P(E) = \frac{1}{4}$

$P(F) = \frac{1}{2}$

$P(E \text{ and } F) = \frac{1}{8} = P(E \cap F)$

(i) $P(E \text{ or } F) = P(E \cup F)$

$= P(E) + P(F) - P(E \cap F)$

$= \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2+4-1}{8}$

$P(E \cup F) = 5/8$

(ii) $P(\underbrace{\text{not } E} \text{ and } \underbrace{\text{not } F})$

$= P(E' \cap F')$

$= P[(E \cup F)']$ De Mor. law

$= 1 - P(E \cup F)$

$= 1 - 5/8 = \frac{3}{8}$

Q.1[illegible] $P(A) = 0.42$

$P(B) = 0.48$

$P(A \text{ and } B) = 0.16 = P(A \cap B)$

(i) $P(\text{not } A) = P(A')$

$= 1 - P(A)$

$= 1 - 0.42 = 0.58$

(ii) $P(\text{not } B) = P(B')$

$= 1 - P(B)$

$= 1 - 0.48 = 0.52$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \text{ or } B)$

$= 0.42 + 0.48 - 0.16$

$= 0.90 - 0.16$

$= 0.74$ ✓

Q.16 $P(\text{not } E \text{ or not } F) = 0.25$

Mutually exclusive Events.

Yes? $P(E \cap F) = 0$

No? $P(E \cap F) \neq 0$

$P(E' \cup F') = 0.25$

$\Rightarrow P(E \cap F)' = 0.25$

$\Rightarrow 1 - P(E \cap F) = 0.25$

$\Rightarrow 1 - 0.25 = P(E \cap F)$

$\Rightarrow \underbrace{P(E \cap F) = 0.75 \neq 0}$

Not M.E.E.

Q.18

40% ⟶ Maths (M) → $P(M) = 0.4$

30% ⟶ Bio (B) ⟶ $P(B) = 0.3$

10% ⟶ Both $(B \cap M) \to P(B \cap M) = 0.1$

$$P(M \text{ or } B) = P(M \cup B) = P(M) + P(B) - P(M \cap B)$$

$$P(M \cup B) = 0.4 + 0.3 - 0.1$$

$$= 0.6$$

Q.19

$P(E_1) = 0.8$

$P(E_2) = 0.7$

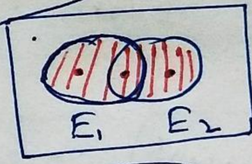
कम से कम

P(passing atleast one of them) = 0.95

① ②

P (Both) = ?

$P(E_1 \cap E_2) = ?$



$E_1 \cup E_2$

$$P(E_1 \cup E_2) = 0.95$$

$$\Rightarrow P(E_1) + P(E_2) - P(E_1 \cap E_2) = 0.95$$

$$\Rightarrow 0.8 + 0.7 - P(E_1 \cap E_2) = 0.95$$

$$\Rightarrow \boxed{P(E_1 \cap E_2) = 0.55}$$

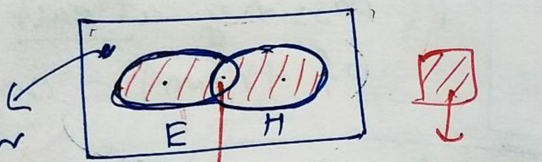
Q.20

$P(E \cap H) = 0.5$

$P(\text{neither}) = \underline{0.1 = P((E \cup H)')}$

$P(E) = 0.75$

$P(H) = ?$



neither

0.1

$(E \cup H)'$

$E \cap H$

$\underline{E \cup H}$

$P((E \cup H)') = 0.1$

$\Rightarrow 1 - P(E \cup H) = 0.1$

$\Rightarrow 1 - 0.1 = P(E \cup H)$

$\Rightarrow P(E \cup H) = 0.9$

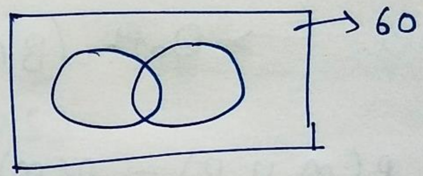
$\Rightarrow P(E) + P(H) - P(E \cap H) = 0.9$

$\Rightarrow 0.75 + \underline{P(H)} - 0.5 = 0.9$

$\Rightarrow 0.25 + \underline{P(H)} = 0.9$

$\Rightarrow \boxed{P(H) = 0.65}$

(21)



NSS → (S)

NCC → (C)

total students $= 60 = n(T)$

$n(S) = 32, \; n(C) = 30$

$n(S \cap C) = 24$

$P(S) = \dfrac{n(S)}{n(T)} = \dfrac{32}{60}$ ✓

$P(C) = \dfrac{n(C)}{n(T)} = \dfrac{30}{60}$ ✓

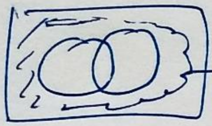
$P(S \cap C) = \dfrac{n(S \cap C)}{n(T)} = \dfrac{24}{60}$ ✓

(i) Prob. of NCC or NSS

$$P(C \cup S) = P(C) + P(S) - P(C \cap S)$$

$$= \frac{30}{60} + \frac{32}{60} - \frac{24}{60} = \frac{38}{60}$$

(ii) Prob. of neither NCC nor NSS $\quad \underline{P(C' \cap S')} = \underline{P((C \cup S)')}$



$$\underline{P((C \cup S)')} = 1 - P(C \cup S)$$

$$= 1 - \frac{38}{60} = \frac{22}{60}$$

(iii) Prob of NSS but $\underline{\text{not NCC}}$.

$$P(S \cap C') = P(S - C)$$

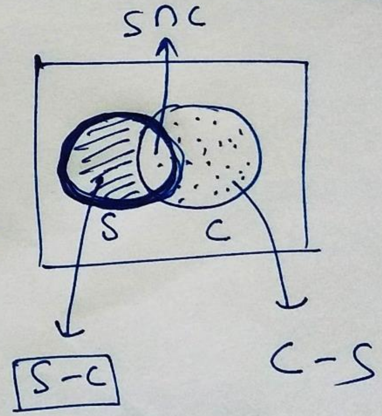
$$= P(S) - P(S \cap C)$$

$$= \frac{32}{60} - \frac{24}{60}$$

$$= 8/60$$

?

$$P(S) = P(S-C) + P(S \cap C)$$



Miscellaneous Exercise - 14.4

- $^nC_r = \frac{n!}{(n-r)!\,r!}$ = (no. of ways to select r objects out of n- different objects.)

- **Multiplication Rule of Counting**

$$\underbrace{\text{Project}}_{} \longrightarrow m \times n$$

No. of ways

$\boxed{m}$ and $\boxed{n}$

Step -I ✓ Step.II ✓

$$P = \frac{\text{fav.}}{\text{total}}$$

$\boxed{\text{Q.1}}$

10 red
20 Blue
30 Green

$\longrightarrow$ 5 Balls

Total no. of cases $= {}^{60}C_5$ ← total

(i) all are blue.

$5 \rightarrow$ Blue ← total Blue 20

fav. cases $= {}^{20}C_5$

$$\boxed{P(\text{all blue}) = \frac{{}^{20}C_5}{{}^{60}C_5}}$$

(ii) atleast one green.

5 Balls

5 0 , 0 G ×
4 0 , 1 G
3 0 , 2 G
2 0 , 3 G
1 0 , 4 G
0 0 , 5 G

total

P(atleast one green)

= 1 − P(no green)

0 Green

total prob.

$= 1 - \dfrac{{}^{30}C_5}{{}^{60}C_5}$ ← total

from Red → 10
& Blue → 20

Q.2

52 Playing Card → 4 cards

3 Diamond & 1 spade

Total no. of ways $= {}^{52}C_4$

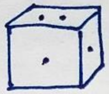
3 Diamond & 1 spade

total ♢ → ${}^{13}C_3$ × ${}^{13}C_1$ 13 → ♠

Fav. Cases $= \left({}^{13}C_3 \times {}^{13}C_1\right)$

$P(3D \& 1S) = \dfrac{{}^{13}C_3 \times {}^{13}C_1}{{}^{52}C_4}$

$\boxed{Q.3}$ Die (पासा)



$\{\underline{1,1}, \underline{2,2,2}, \underline{3}\}$

(i) $P(2) = \dfrac{3}{6}$ ← fav. ② ← total

$= \dfrac{1}{2}$

(ii) $P(1 \text{ or } 3) = \dfrac{3}{6} = \dfrac{1}{2}$

$\{1,1\}$ $\{3\}$

(iii) $P(\text{not } 3) = \dfrac{5}{6}$

$\{1,1,2,2,2\}$

$\boxed{Q.4}$

total tickets ← 10000

9990 Not Prize (win)	10 Prize

$P(\text{not winning}) = ?$

$^nC_r = \dfrac{n!}{(n-r)!\,r!}$

$^nC_1 = n$

ⓐ if we ~~by~~ buy 1 ticket

$\text{Total} = {}^{10000}C_1 = 10000$

$\text{favourable (not Prize)} = {}^{9990}C_1 = 9990$

$P = \dfrac{999\cancel{0}}{1000\cancel{0}} = \dfrac{999}{1000}$

(b) if we buy 2 tickets

total = $^{10000}C_2$

fav. = $^{9990}C_2$

P(not winning) = $\frac{^{9990}C_2}{^{10000}C_2}$ ✓

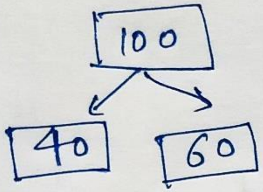
(c) if we ~~b~~ buy 10 tickets.

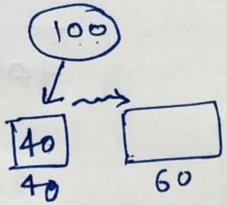
total = $^{10000}C_{10}$

fav. = $^{9990}C_{10}$

P(not winning) = $\frac{^{9990}C_{10}}{^{10000}C_{10}}$ ✓

Q.5



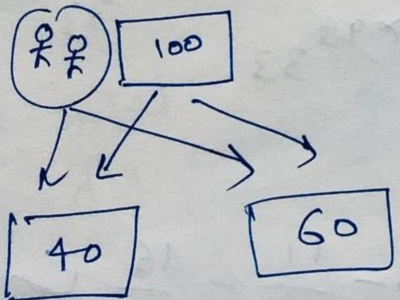


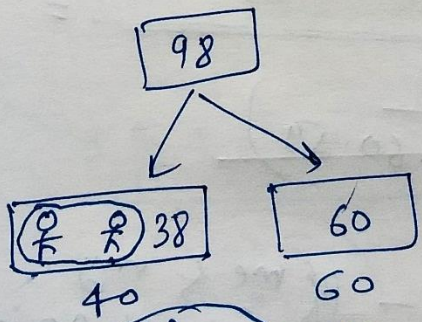
total $= {}^{100}C_{40}$ या ${}^{100}C_{60} = \dfrac{100!}{40!\,60!}$

selection

$\boxed{\text{Property } {}^nC_r = {}^nC_{n-r}}$

(i) me & my friend are in Same Section.

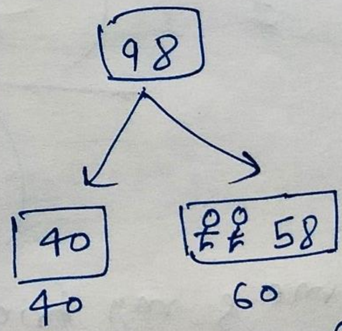




+

or

या



${}^{98}C_{38} = {}^{98}C_{60} + {}^{98}C_{40} =$ fav. Cases

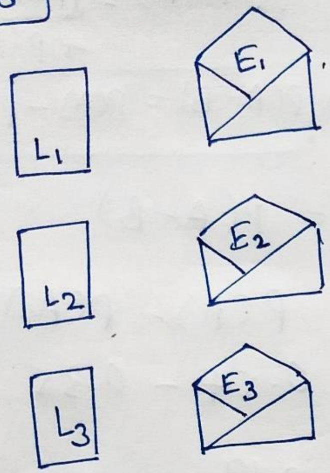
P(me & my friend are is same section) $= \dfrac{{}^{98}C_{38} + {}^{98}C_{40}}{{}^{100}C_{40}} = \dfrac{\left(\dfrac{98!}{38!\,60!}\right) + \left[\dfrac{98!}{40!\,58!}\right]}{\left[\dfrac{100!}{40!\,60!}\right]}$

$$= \frac{\left[\dfrac{98!}{38!\,60.59.58!}\right] + \left[\dfrac{98!}{40.39.38!58!}\right]}{\left[\dfrac{100\times 99\times 98!}{40.39.38!\,60.59.58!}\right]} = \frac{\cancel{\dfrac{98!}{38!\,58!}}\left[\dfrac{1}{60.59} + \dfrac{1}{40.39}\right]}{\cancel{\dfrac{98!}{38!\,58!}}\cdot\left[\dfrac{100\times 99}{40.39.60.59}\right]}$$

$$= \frac{\left(\dfrac{40.39 + 60.59}{\cancel{60.59.\,40.39}}\right)}{\left(\dfrac{100\times 99}{\cancel{40.39.60.59}}\right)} = \frac{\overset{17}{\cancel{5100}}}{\underset{33}{\cancel{100\times 99}}} = \frac{17}{33}$$ ✓

(b) P(me & my friend) go in different section $= 1 - P\left(\begin{array}{c}\text{me \& my}\\ \text{friend go in}\\ \text{same section}\end{array}\right) = 1 - \dfrac{17}{33} = \dfrac{16}{33}$ ✓

Q.6



1 letter → 1 enevelope

Total randomly fill $= 3 \times 2 \times 1 = 6$

Answer.

$P = \dfrac{4}{6}$ ← fav. / ← total

$= \dfrac{2}{3}$ ✓

Favourable Cases.

atleast one letter into proper envelope

कम से कम

1 letter → Proper Envelope

$L_1 \to E_1$, L_2, L_3, E_2, E_3 या $L_2 \to E_2$ या $L_3 \to E_3$

3 cases.

2 letters → Proper Envelope या 3 letter

$L_1 \to E_1$
$L_2 \to E_2$
$L_3 \to E_3$

1 case

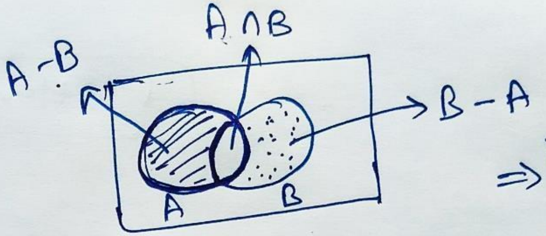
∴ total fav. cases = 4

Q.7

$P(A) = 0.54$

$P(B) = 0.69$

$P(A \cap B) = 0.35$



$\therefore P(A) = \boxed{P(A-B)} + P(A \cap B)$

$\Rightarrow \boxed{P(A-B) = P(A) - P(A \cap B)}$

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.54 + 0.69 - 0.35$

$= 0.54 + 0.34 = 0.88$

(ii) $P[A' \cap B'] = P[(A \cup B)']$

(De-Morgan's Law)

$= 1 - P(A \cup B)$

$= 1 - 0.88$

$= 0.12$

(iii) $P(A \cap B') = P(A-B)$

$= P(A) - P(A \cap B)$

$= 0.54 - 0.35$

$= 0.19$

(iv) $P(B \cap A') = P(B-A)$

$= P(B) - P(A \cap B)$

$= 0.69 - 0.35$

$= 0.34$

Q.8

either — or —

$P(\underset{A}{\underline{\text{male}}} \cup \underset{B}{\underline{\text{over 35 years}}}) = P(\text{male}) + P(\text{over 35 years})$

$- P(\underline{\text{male}} \cap \underline{\text{over 35 years}})$

$P(\text{male}) = \dfrac{3}{5}$ ← male, ← total

$P(\text{over 35 years}) = \dfrac{2}{5}$

$P(\underline{\text{male}} \cap \underline{\text{over 35 years}}) = \dfrac{1}{5}$

↓ both

put

$= \dfrac{3}{5} + \dfrac{2}{5} - \dfrac{1}{5}$

$= \dfrac{4}{5}$

Q. 9 (i) Digits are repeated

4 digit > 5000

0/1/3/5/7

total

5 _ _ _

No. of ways → 5 . 5 . 5 $= 5.5.5-1$ (5000 वाला)

$= 125-1$

$= 124$

0/1/3/5/7

7 _ _ _

No. of ways → 5 . 5 . 5 $= 5.5.5$

$= 125$

total = 249

P(No. divisible by 5) $= \frac{\text{fav.}}{\text{total}} = \frac{99}{249} = \frac{33}{83}$

fav. Cases → No. divisible by 5 → 0/5 last Digit

0/1/3/5/7 0/5

5 _ _ _

5 5 2 $= 5.5.2-1$ (5000 वाला)

$= 49$

0/5

7 _ _ _

5 . 5 . 2 $= 50$

total fav. = 99

Q.9 (ii) The repitition of digits is not allowed.

4 digit > 5000

0 | 1 | 3 | 5 | 7

total.

0 | 1 | 3 | 7

5 _ _ _

No. of ways → 4 . 3 . 2 = 24

0 | 1 | 3 | 5

7 _ _ _

No. of ways → 4 . 3 . 2 = 24

total = 48

P(No. is divisible by 5) = $\frac{\text{fav.}}{\text{total.}} = \frac{\cancel{18}\,3}{\cancel{48}\,8} = \frac{3}{8}$

Repitition is not allowed

Fav. cases → No. is divisible by 5 → last Digit 0/5

1/3/7 | 0

| 5 | _ | _ | _ |

3 . 2 . 1 = 6

0/5

| 7 | _ | _ | _ |

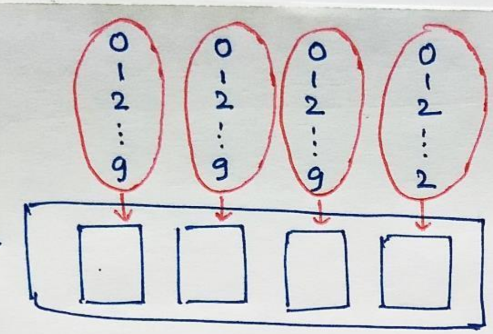
3 . 2 . 2 = 12

1/3/Remaining

total fav. = 18

Q.10

Lock →



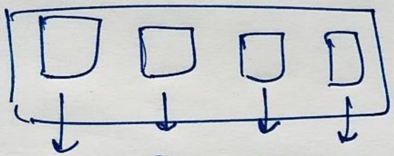
No repeats

$$P\left(\text{get the right sequence to open the lock}\right) = \frac{1}{5040}$$

↙ fav.

↖ total

Total =



$10 \times 9 \times 8 \times 7$

0 | 1 | 2 | ... | 9

total $= 10 \times 9 \times 8 \times 7$

$= 10 \times 9 \times 56$

$= 5040$

Note: To open a lock, we always have only one code (password).

$\therefore$ ~~Favoura~~ $\boxed{\text{No. of favourable cases} = 1}$